

# Galactic rotation curves and strange quark matter with observational constraints

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## Abstract

We obtain the space time of the galactic core in the framework of general relativity by taking the flat rotational curve as input and considering the matter content in the galactic core region as strange quark matter. We also obtain the energy density, radial and tangential pressures of the core strange quark matter. Significantly we have shown that Bag Constant takes an important role to stabilize the circular orbit of the test particles. We also give a limit of the Bag Constant for the existence of quark matter in the galactic halo region.

## 1 Introduction

Presence of dark matter was suspected first by Oort [1, 2, 3]. Later Zwicky [4, 5] concluded that a large amount of non-luminous matter remains hidden in the galactic haloes. Observation of flatness of galactic rotation curves [6, 7, 8, 9, 10, 11] was consequitively confirmed the Zwicky's suggestion regarding the presence of dark matter. Afterwards, gravitational lensing of objects like bullet clusters and the temperature distribution of hot gas in galaxies and galactic clusters have further confirmed the existence of dark matter. The flatness of the galactic rotation curves indicates the presence of much more

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matter within the galaxy than the visible material. Estimation supports a spherical distribution of matter (galaxy) is surrounded by a dark matter halo. Gravitational effect of dark matter is more manifest at larger radius[12]. Now a days, the amount of dark matter present in the Universe has become known in more precise manner. Cosmic Microwave Background Radiation data indicates that nearly 85% of total matter in the galaxy is dark in nature. Big Bang analysis and cosmological observational data indicates that the bulk of dark matter may be cold or warm, stable or long-lived and non-interacting with the visible matter. However, after a long and through investigations, a little is yet known about the nature of dark matter.

Gell-Mann[13] and Zweig [14] had independently proposed that hadrons are consists of even more fundamental particles called quarks, a proposition which got experimental support later on. As quarks are not free particles, the quark confinement mechanism have been dealt with great details in Quantum Chromo Dynamics (QCD). In the MIT bag model[15], it was suggested that the quark confinement is due to a universal pressure  $B_g$ , which is called the bag constant. Farhi and Jafee [16] and Alcock et al [17] had shown that the value of the bag constant,  $B_g$  should lie between  $60-100 \text{ MeV}/fm^3$ , for a strange quark matter.

There are a number of proposed candidates for dark matter. One of them is standard cold dark matter (SCDM)[18, 19]. In this literature, we suggested that quark matter may be such candidate which is also proposed by several researchers[20, 21, 22]. Also, it is well accepted that quark matter is exist at the centre of neutron stars, strange stars[20, 23, 24, 25, 26, 27, 28]. Immediately after the Big Bang the Universe underwent a Quark-Gluon-Plasma (QGP) phase. In the famous Large Hadronic Collider (LHC) experiment, Scientists recreate the conditions similar to those encountered before and in the early hadronization period[29]. As the expansion takes place, the Universe cools down and the hot Quark-Gluon-Plasma (QGP) freezes slowly to produce individual hadrons[30]. Most acceptable theory of strong interaction, Quantum Chromo Dynamics (QCD), suggests that under extreme condition a hadronic system can undergoes a phase transition from confined hadronic matter to the QGP phase. It is suggested that the core of neutron stars may consists of cold QGP[31].

The purpose of the present work is to show that, with the input of flat rotation curve and assuming the dark matter contents as strange quark matter, the gravity in the halo region is attractive under certain condition where Bag Constant takes an important role. We also want to investigate the presence of quark matter in the galactic halo region. This is the main motto of our work.

## 2 Field equations and general results

The general static spherically symmetric spacetime is represented as

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

where  $\nu(r)$  and  $\lambda(r)$  are the metric potentials and are function of the space coordinate  $r$  only. We further assume that the energy-momentum tensor for the strange quark matter filling the interior of the galaxy may be expressed in the standard form as

$$T_{ij} = \text{diag}(\rho, -p_r, -p_t, -p_t)$$

where  $\rho$ ,  $p_r$  and  $p_t$  correspond to the energy density, radial pressure and transverse pressure of the baryonic matter, respectively.

The Einstein's field equations for the metric (1) in presence of strange quark matter are then obtained as (with  $G = c = 1$  under geometrized relativistic units)

$$e^{-\lambda} \left( \frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = 8\pi\rho, \quad (2)$$

$$e^{-\lambda} \left( \frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = 8\pi p_r, \quad (3)$$

$$\frac{1}{2}e^{-\lambda} \left( \nu'' + \frac{\nu'^2}{2} - \frac{\nu'\lambda'}{2} + \frac{\nu' - \lambda'}{r} \right) = 8\pi p_t \quad (4)$$

Following the MIT bag model, we express the strange quark matter equation of state(EOS) in the form

$$p_r = \frac{1}{3}(\rho - 4B_g), \quad (5)$$

where  $B_g$  is the Bag constant(in  $MeV/fm^3$  units).

## 3 Solutions for the strange quark matter model

Assuming the known flat rotation curve condition  $v_\phi = [\frac{r(e^\nu)'}{2e^\nu}]^{1/2} = \text{constant}$  tangential velocity[33, 32] obtained a solution of it as [34]

$$e^\nu = B_0 r^l, \quad (6)$$

where  $l = 2v_\phi^2$  and  $B_0$  is an integration constant.

It is to be noted that the observed rotational velocity  $v^\phi$  becomes more or less a constant with  $v^\phi \sim 10^{-3}(300 \text{ km/s})$  for a typical galaxy [35, 36].

Now, from equations (2)-(5) and then using the simple expression in the equation(6) we get the simplified form as

$$-(e^{-\lambda}\lambda') + (4 + 3l)\frac{e^{-\lambda}}{r} = \frac{4}{r} - 32\pi B_g r \quad (7)$$

Substituting  $e^{-\lambda} = x$ , we get

$$x' + \frac{ax}{r} = \frac{4}{r} + cr \quad (8)$$

where  $a = 4 + 3l, c = -32\pi B_g$ . Solving the above equation we get the metric potential as

$$e^{-\lambda} = \frac{4a + car^2 + 8}{a(a + 2)} + \frac{D}{r^a} \quad (9)$$

where D is integration constant.

Therefore, with the flat rotational curve condition, the metric (1) becomes

$$ds^2 = -B_0 r^l dt^2 + \left[ \frac{4a + car^2 + 8}{a(a + 2)} + \frac{D}{r^a} \right]^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (10)$$

Using equation (6) and (9) in (2)-(5), we get

$$\rho = \frac{1}{8\pi} \left( \frac{D(a - 1)}{r^{a+2}} - \frac{3c}{a + 2} + \frac{a - 4}{ar^2} \right) \quad (11)$$

$$p_r = \frac{1}{8\pi r^2} \left[ \left( \frac{4a + 8 + car^2}{a(a + 2)} + \frac{D}{r^a} \right) (l + 1) - 1 \right] \quad (12)$$

$$p_t = \frac{1}{8\pi} \left[ \left( \frac{l}{2r} \right)^2 \left( \frac{4a + 8 + car^2}{a(a + 2)} + \frac{D}{r^a} \right) + \left( \frac{l}{4} + \frac{1}{2} \right) \left( \frac{2c}{a + 2} - \frac{aD}{r^{a+2}} \right) \right] \quad (13)$$

The pressure anisotropy is a good feature from the point of view of exterior matching. The solution can be matched to the Schwarzschild exterior metric at the boundary of the halo[38]. As the strange matters are not exotic in nature, they must satisfies the Null Energy Condition(NEC) [39]. We find that the NEC is satisfies at everywhere i.e.  $\rho > 0$  and  $\rho + p_r > 0$ . However, we note that here the Strong Energy Condition(SEC) is violated ( see figure 1). Therefore, we comment the quark matter in the galactic halo region is marginally real like.

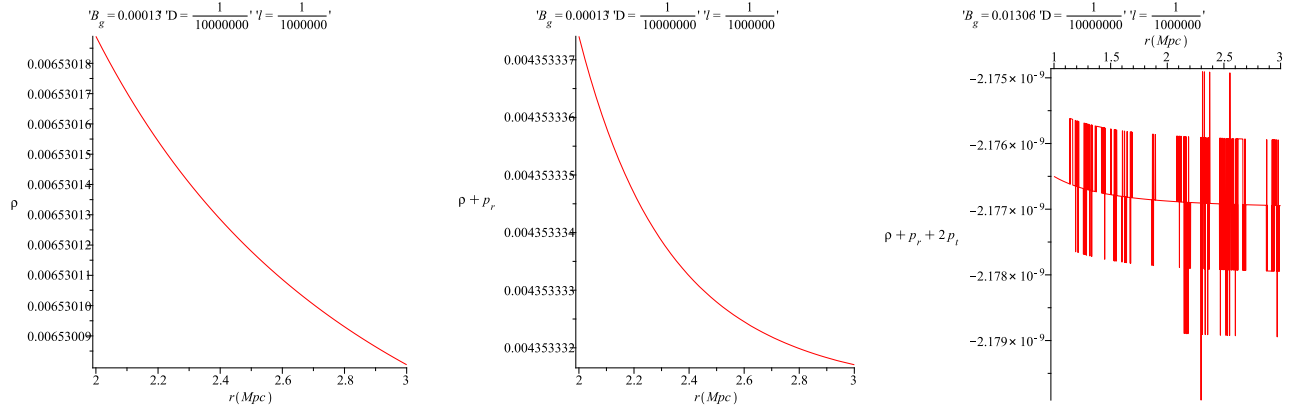


Figure 1: (Left) Variation of  $\rho$  with distance  $r$ (Mpc) for the specific values of the parameters,  $D = 10^{-7}$ ,  $l = 10^{-6}$ ,  $B_g = .00013$ . (Middle) Variation of  $\rho + p_r$  with distance  $r$ (Mpc) for the same specific values of the parameters. (Right) Variation of  $\rho + p_r + 2p_t$  with distance  $r$ (Mpc) for the same specific values of the parameters.

## 4 Few Features

### 4.1 Stability

Let, the four velocity,  $U^\alpha = \frac{dx^\sigma}{d\tau}$  for a test particle moving solely in the galactic halo region (restricting ourself to  $\theta = \pi/2$ ), the equation  $g_{\nu\sigma}U^\nu U^\sigma = -m_0^2$  can be cast in a Newtonian form as[37]

$$\left(\frac{dr}{d\tau}\right)^2 = E^2 + V(r) \quad (14)$$

which gives

$$V(r) = - \left[ E^2 \left( 1 - \frac{r^{-l} \left[ \frac{4a+8+car^2}{a(a+2)} + \frac{D}{r^a} \right]}{B_0} \right) + \frac{4a+8+car^2}{a(a+2)} + \frac{D}{r^a} \left( 1 + \frac{L^2}{r^2} \right) \right] \quad (15)$$

$$E = \frac{U_0}{m_0}, L = \frac{U_3}{m_0} \quad (16)$$

where the constants  $E$  and  $L$ , respectively, represents the conserved relativistic energy and angular momentum per unit mass of the test particle. Circular orbits of the test particles are defined by  $r = R = \text{constant}$ , so that  $\frac{dR}{d\tau} = 0$  and  $\frac{dV}{dr}|_{r=R} = 0$ . From these two conditions we get the conserved parameters as:

$$L = \pm \sqrt{\frac{l}{2-l}} R \quad (17)$$

and using this in  $V(R) = -E^2$ , one can easily get,

$$E = \pm \sqrt{\frac{2B_0}{2-l}} R^{\frac{l}{2}} \quad (18)$$

For stable orbit  $\frac{d^2V}{dr^2}|_{r=R} < 0$  and for unstable orbit  $\frac{d^2V}{dr^2}|_{r=R} > 0$ . Putting the expressions for L and E in  $\frac{d^2V}{dr^2}|_{r=R}$ , we get the final results as

$$\frac{d^2V}{dr^2}|_{r=R} = - \left[ \frac{2l}{R^2} \left( \frac{4a+8+caR^2}{a(a+2)} + \frac{D}{R^a} \right) + \frac{2(2l-1)}{R(2-l)} \left( \frac{aD}{R^{a+1}} - \frac{2cR}{a+2} \right) \right] \quad (19)$$

Thus  $\frac{d^2V}{dr^2}|_{r=R} < 0$  so that circular orbits are stable under consideration when  $D > 0$  and  $B_g < \frac{(a+2)(2-l)}{64\pi(2-2l-l^2)} \left[ \frac{8l}{aR^2} + \frac{2D}{R^{a+2}} \left( l - \frac{a(1-2l)}{2-l} \right) \right]$ . The first condition is always satisfied since, from equation(10), we see that D has the same dimension as radius, R. Again, as the Bag Const.,  $B_g$  and radius R are always positive, so D is obviously positive. As this results is in agreement with the observations, therefore we can conclude that the value of the Bag constant,  $B_g$  should be less than  $\frac{(a+2)(2-l)}{64\pi(2-2l-l^2)} \left[ \frac{8l}{aR^2} + \frac{2D}{R^{a+2}} \left( l - \frac{a(1-2l)}{2-l} \right) \right]$ , if indeed, there exists any quark matter present in the galactic haloes. Therefore from this model, we can give the limit and a rough estimation of the Bag constant within the galactic halo region. Thus we note that Bag Const.,  $B_g$  plays an important role to stable the circular orbit.

## 4.2 Attraction in Strange quark matter

Observational indication is that the gravity on the galactic scale is attractive. Again, the stable circular orbit indicates that the particles are being accelerated towards the Centre of the Galaxy. We can see it by studying the geodesic for a test particle that has been placed at a circular path of radius  $r$ .

Now,

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma_{\alpha}^{\mu\gamma} \frac{dx^\mu}{d\tau} \frac{dx^\gamma}{d\tau} = 0. \quad (20)$$

This equation implies that

$$\frac{d^2r}{d\tau^2} = -\frac{1}{2} \left[ \frac{4a+8+car^2}{a(a+2)} + \frac{D}{r^a} \right] \left[ \left( \frac{Da}{r^{a+1}} - \frac{2car}{a(a+2)} \right) \left( \frac{4a+8+car^2}{a(a+2)} + \frac{D}{r^a} \right)^{-2} \left( \frac{dr}{d\tau} \right)^2 + B_0 l r^{l-1} \left( \frac{dt}{d\tau} \right)^2 \right]. \quad (21)$$

Obviously, the quantities in the square brackets are positive. Thus particles are attracted towards the centre with some boundaries.

### 4.3 Gravitational Energy

We can easily determine the total gravitational energy  $E_g$  between two fixed radii, say,  $r_1$  and  $r_2$  [45]:

$$\begin{aligned} E_g &= M - E_M = 4\pi \int_{r_1}^{r_2} [1 - \sqrt{e^{\lambda(r)}}] \rho r^2 dr \\ &= 4\pi \int_{r_1}^{r_2} \left[ 1 - \sqrt{\frac{1}{\frac{4a+car^2+8}{a(a+2)} + \frac{D}{r^a}}} \right] \left[ \frac{1}{8\pi} \left( \frac{D(a-1)}{r^{a+2}} - \frac{3c}{a+2} + \frac{a-4}{ar^2} \right) \right] r^2 dr. \end{aligned} \quad (22)$$

where

$$M = 4\pi \int_{r_1}^{r_2} \rho r^2 dr \quad (23)$$

is the Newtonian mass given by

$$M = 4\pi \int_{r_1}^{r_2} \rho r^2 dr = \frac{1}{2} \left[ \frac{Da}{(2-a)r^{a-2}} - \frac{D}{(1-a)r^{a-1}} + \frac{a-4}{a}r - \frac{cr^3}{a+2} \right]_{r_1}^{r_2} \quad (24)$$

Here,  $E_M$  is the sum of the other forms of energy like the rest energy, kinetic energy, and internal energy.

For the specific values of the parameters  $r_1 = 1$ ,  $r_2 = 3$ ,  $D = 10^{-7}$ ,  $l = 10^{-6}$ ,  $B_g = .00013$ , we calculate the numerical value of the integrand (22) describing the total gravitational energy as  $E_g = -0.00168098$ , which indicates that  $E_g < 0$ , in other words, there is an attractive effect in the halo region. This result is very much expected for the matter source in the galactic halo region that produces stable the circular orbit.

### 4.4 Observational Constraints

We can rewrite the metric(1) in the form

$$ds^2 = -e^{2\Phi(r)} dt^2 + \frac{1}{1 - \frac{2m(r)}{r}} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (25)$$

where, the metric functions are given by

$$\Phi(r) = \frac{1}{2} (\ln B_0 + l \ln r) \quad (26)$$

$$m(r) = \frac{r}{2} \left[ 1 - \frac{4a + car^2 + 8}{a(a+2)} - \frac{D}{r^a} \right] \quad (27)$$

These two functions are not necessarily the same as the potential and mass functions are found from the observations. The first post-Newtonian approximation [40], the gravitational potential  $\Phi(r)$ , in general relativity, is given by

$$\nabla^2\Phi \approx R_{tt} \approx 4\pi(\rho + p_r + 2p_t). \quad (28)$$

which reduces to the equation

$$\nabla^2\Phi_N = 4\pi\rho \quad (29)$$

provided the contributions of the pressures are negligible in comparison to energy density. Here,  $\Phi_N$  indicates the Newtonian potential. This general relativistic  $\Phi$  also appears in the wavelength shifts  $z_{\pm}$  of an emission line of a massive particle for an edge-on galaxy is [41, 42] is given by

$$1 + z_{\pm} = \frac{1}{\sqrt{1 - r\Phi'(r)}} \left[ \frac{1}{e^{\Phi(r)}} - \frac{\pm |b| \sqrt{r\Phi'(r)}}{r} \right] \quad (30)$$

where  $\pm$  signs indicates the approaching and receding particles and  $b$  is the impact parameter. The above equation(30) can approximately written as [43]  $z_{\pm}^2 \approx r\Phi'(r)$ .

Therefore, the usual methods for obtaining potential for rotation curve measurements yields the pseudo-potential

$$\Phi_{RC} = \Phi \neq \Phi_N. \quad (31)$$

From equation(28) one can say that pseudo-mass,

$$m_{RC} = r^2\Phi'(r) \approx 4\pi \int (\rho + p_r + 2p_t)r^2dr = \frac{lr}{2} \quad (32)$$

The pseudo-mass  $m_{RC}$  becomes Newtonian mass  $M(r)$  when pressure contributions are negligible in comparison to energy density.

We can also treat Photons as probe particles as they interacts with the gravitational field in the core of the galaxy during their travel to the observer. The gravitational effect on photon motion can be measured in terms of a refractive index  $\mu(r)$ [44]. The geodesic of both massive and photonlike particles could be expressed exactly in terms of a single generalized refractive index  $N = \frac{\mu^2 v}{c}$  where  $v$  is the three-velocity of the particle. For light motion,  $v = \frac{c}{\mu}$ .

For unspecified metric functions  $\Phi(r)$  and  $m(r)$ , [43] argue that  $\mu(r) = 1 - 2\Phi_{lens} + O[\Phi_{lens}^2]$  where they defined the lensing pseudo-potential as

$$\Phi_{lens} = \frac{1}{2}\Phi(r) + \frac{1}{2} \int \frac{m(r)}{r^2} dr. \quad (33)$$

Another pseudo-mass obtained from lensing measurement has been written as [43]

$$m_{lens} = \frac{1}{2}r^2\Phi'(r) + \frac{1}{2}m(r). \quad (34)$$



The first order approximation of Einstein's equation yield

$$\rho(r) \approx \frac{1}{4\pi r^2} [2m'_{lens}(r) - m'_{RC}(r)] \quad (35)$$

$$4\pi r^2(p_r + 2p_t) \approx 2[m'_{RC}(r) - m'_{lens}(r)] \quad (36)$$

where the right-hand sides represents, respectively, pseudo-density and pseudo-pressures. Furthermore, [43] defined a dimensionless quantity

$$\omega(r) = \frac{p_r + 2p_t}{3\rho} \approx \frac{2(m'_{RC} - m'_{lens})}{3(2m'_{lens} - m'_{RC})}. \quad (37)$$

where the pseudo-quantities on the right hand side of the equations (35-37) determine, respectively, the observed density, pressure and equation of state.

It is to be mentioned that the observable quantities are the pseudo-quantities. The general procedure to determine the metric are as follows: Once anyone is able to observationally determine the profiles of pseudo-quantities, one can work backwards to find the corresponding metric functions. This kind of reverse technique observational astrophysicists use. If the observed pseudo-profiles fit (up to experimental error) with the analytic profiles of a priori given metric functions, one can say that the solution is physically sustainable. Otherwise, it has to be ruled out as non-viable. In this sense, the observed pseudo-profiles play the role of constraints on the possible metric solutions.

For the present situations, we obtain the following constraint equations on  $\Phi(r)$ ,  $m(r)$ , the pressure profile and the equation of state:

$$\Phi_{RC} = \frac{1}{2} (\ln B_0 + l \ln r) \quad (38)$$

$$m_{RC} = r^2 \Phi'(r) = \frac{lr}{2} \quad (39)$$

$$\Phi_{lens} = (l + 1 - \frac{4}{a}) \frac{\ln r}{4} + \frac{1}{4} \ln B_0 + \frac{r^2}{4a} \left[ \frac{D}{r^{a+2}} - \frac{ac}{2(a+2)} \right] \quad (40)$$

$$m_{lens} = \frac{l^2}{8} + \frac{r}{4} \left( 1 - \frac{4a+8+car^2}{a(a+2)} - \frac{D}{r^a} \right) \quad (41)$$

$$2(m'_{RC} - m'_{lens}) = 2 \left[ \frac{l}{2} - \frac{1}{4} \left( 1 - \frac{4a+8+car^2}{a(a+2)} - \frac{D}{r^a} \right) - \frac{r}{4} \left( -\frac{2car}{a(a+2)} + \frac{Da}{r^{a+1}} \right) \right] \quad (42)$$

$$\omega(r) = \frac{2(m'_{RC} - m'_{lens})}{3(2m'_{lens} - m'_{RC})} = \frac{2 \left[ \frac{l}{2} - \frac{1}{4} \left( 1 - \frac{4a+8+car^2}{a(a+2)} - \frac{D}{r^a} \right) - \frac{r}{4} \left( -\frac{2car}{a(a+2)} + \frac{Da}{r^{a+1}} \right) \right]}{6 \left[ \frac{1}{4} \left( 1 - \frac{4a+8+car^2}{a(a+2)} - \frac{D}{r^a} \right) + \frac{r}{4} \left( -\frac{2car}{a(a+2)} + \frac{Da}{r^{a+1}} \right) \right]} - \frac{3l}{2} \quad (43)$$

It is to be noted that the pseudoquantities given in Eqs. (38)-(43) are actual observables from the combined measurements of rotation curves and gravitational lensing. Equation (43) provides a convenient parameter that gives a 'measure' of the equation of state of the halo field, calculated from a combination of rotation curves and lensing measurements. Figure 2 shows that the value of  $\omega$  is negative indicating repulsion.

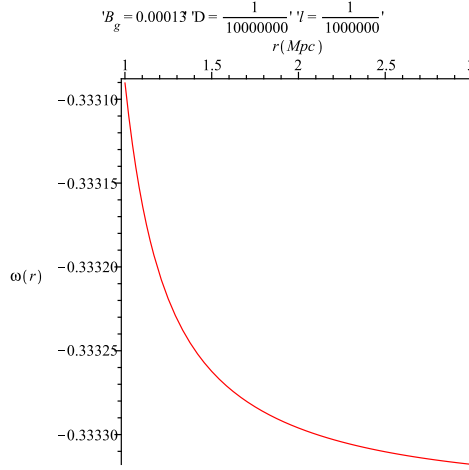


Figure 2: The variation  $\omega(r)$  with respect to  $r$ .

## 5 Conclusions

We obtain the space time of the galactic core in the framework of general relativity by assuming the flat rotational curve condition and considering the matter content in the galactic core region as strange quark matter. The spacetime metric is neither asymptotically flat nor a spacetime due to a centrally symmetric black hole. We also obtain the energy density, radial and tangential pressures of the core strange quark matter. We see that the NEC is satisfied. However, we note that the SEC is violated. Therefore, it is obvious that quark matter is not purely real like and it behaves like dark matter. Significantly we have shown that the Bag Constant takes an important role to stabilize the circular orbit of the test particles. We also give a limit of the Bag Constant for the existence of quark matter in the galactic halo region.

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